The University of Nottingham

DEPARTMENT OF MECHANICAL, MATERIALS AND MANUFACTURING ENGINEERING

A LEVEL 2 MODULE, AUTUMN SEMESTER 2014-2015

MECHANICS OF SOLIDS 2

Time allowed TWO Hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced

Answer FOUR questions

Only silent, self contained calculators with a Single-Line Display or Dual-Line Display are permitted in the examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn examination paper over until instructed to do so

Several of our exam papers require students to answer a specific number of questions from those available, e.g. answer 4 out of 6 questions. If students answer more than the required number of questions only the required number will be marked, so for the example given, only the first 4 solutions in the exam script will be marked.

If during the exam students attempt additional questions they should clearly indicate on their script which solutions they want to be marked - simply putting a line through solutions that should be disregarded is recommended.

ADDITIONAL MATERIAL: Graph Paper

INFORMATION FOR INVIGILATORS:

Question papers should be collected in at the end of the exam – do not allow candidates to take copies from the exam room.

1. A sign is supported on a post with an outer diameter of 100mm and in inner diameter 80mm as shown in Figure Q1. A wind pressure of 2kPa acts on the sign.



Figure Q1

- (a) Calculate the resultant force, *W*, applied by the wind pressure on the centre of the sign. [2 marks]
- (b) Calculate the magnitude of the torque and bending moment caused by *W* about point A shown in Figure Q1. [4 marks]
- (c) Determine the state of stress on a plane stress element located at point A, include a sketch. [8 marks]
- (d) Determine the magnitude of the in-plane principal stresses and maximum shear stress at point A, include a sketch of Mohr's Circle. [11 marks]

2. An assembly that consists of a bolt and a sleeve as shown in Figure Q2 is tightened to prevent movement with no pre-stress at a temperature of 25°C. The length of the assembly is 125 mm. The bolt is made from steel with a modulus of elasticity of 200 x 10^9 Pa and has a cross-sectional area of 400 mm²; the sleeve is made from aluminium with a modulus of elasticity of 73.1 x 10^9 Pa and has a cross-sectional area of 600 mm².



Figure Q2

The assembly is subjected to a temperature increase to 90°C. The thermal expansion coefficients of steel and aluminium are 12×10^{-6} /°C and 23×10^{-6} /°C respectively.

- (a) Determine the total displacement of the assembly after the temperature rise. [10 marks]
- (b) Calculate the stresses in the bolt and the sleeve.

[10 marks]

(c) If the length of the assembly is doubled to 250 mm but the temperature rise is the same, determine the stresses in the bolt and the sleeve. [5 marks] 3. A 2m long beam is simply supported at both ends. From the left hand side, it is subjected to a point moment of 0.75kNm a quarter of the way along its length, a point load of 2kN half way along its length and a continuous uniformly distributed load starting three quarters of the way along its length as shown in Figure Q3. The beam is made from Aluminium and has a Young's Modulus, E, of 70GPa and a second moment of area, I, of $1.4 \times 10-7m^4$.



Figure Q3

- (a) Determine the reaction forces at the beam supports. [5 marks]
- (b) Calculate the deflection of the beam at the position of the applied point load. [12 marks]
- (c) Calculate the deflection and the slope of the beam at the position of the applied point moment. [8 marks]

4. A rectangular section beam, 125mm wide and 200mm deep, is loaded in pure bending, about the Y-Y axis as shown in Figure Q4. The material can be assumed to be elastic-perfectly plastic with a yield stress of 200MPa and a Young's modulus of 210GPa.



All dimensions in mm



- (a) Calculate the radius of curvature and bending stress distribution when a moment of 200kNm is applied. [13 marks]
- (b) Calculate the residual radius of curvature and bending stress distribution when the bending moment is removed. [12 marks]

- 5. (a) Describe with the aid of diagrams the relationship between crack growth rate and stress intensity factor. [7 marks]
 - (b) Show with the aid of diagrams the effects of mean stress on fatigue crack growth. [4 marks]



Figure Q5

An aircraft structural component in the form of an aluminium panel as shown in Figure Q5 is subjected to an external load that gives a nominal tensile stress of a half of the yield point. The yield stress and the fracture toughness of the material are 360 MPa and 40 MPa $m^{1/2}$ respectively.

(c) Determine the maximum size of crack or flaw that may exist in the panel shown in Figure Q5 without affecting unstable crack propagation assuming linear elastic material and that:

$$K_I = \sigma_{nom} \sqrt{\pi a}$$

[6 marks]

(d) It is found during the inspection that there is a crack contained in the panel shown in Figure Q5 with the size a = 1 mm. If the external load is fluctuated such that the maximum nominal stress is half of the yield point and the minimum is compressive of 30 MPa, estimate the life cycle of the structure. [8 marks]

6. The Tresca or τ_{max} criterion states that the material will yield if:

 $\sigma_1 - \sigma_3 \ge \sigma_y \qquad \sigma_1 > \sigma_2 > \sigma_3$

The von Mises yield criterion states that the material will yield if:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \ge 2\sigma_v^2$$

- (a) Show the yield boundaries for Tresca and von Mises, for a two-dimensional stress state, on the σ_1 - σ_2 plane (σ_3 =0). [3 marks]
- (b) Represent the yield surfaces for Tresca and von Mises for a threedimensional stress state. [3 marks]
- (c) On (b) show the decomposition of the stress into hydrostatic and deviatoric components. [3 marks]
- (d) The angled bar shown in Figure Q6 is subjected to a load, P, as shown. The yield stress of the material is 250MPa. Considering both the Tresca and von Mises yield criteria, for each case determine the maximum value of P that will just cause yielding to occur in the bar at point A. [16 marks]

